Addition Mistakes (Forensic Accountancy)

This test follows on from the Addition Mistakes project on page 82 of *Proof: Interesting activities in conjecture and mathematical proof*

Name: MARKING SCHEDULE

1. [6 marks] Total marks = 60

The number 34567 is written in expanded form as $3 \times 10\,000 + 4 \times 1000 + 5 \times 100 + 6 \times 10 + 7$

The number 6789 is written in expanded form as $6 \times 1000 + 7 \times 100 + 8 \times 10 + 9$

The number 10203 is written in expanded form as $1 \times 10\,000 + 2 \times 100 + 3$

Write these numbers in expanded form:

(a) 234 $2 \times 100 + 3 \times 10 + 4$ ✓✓

(b) 67 089 $6 \times 10000 + 7 \times 1000 + 8 \times 10 + 9$ ✓✓

(c) 65 $6 \times 10 + 5$ ✓✓

2. [8 marks] In this question each letter stands for a digit, for example if $a = 3$, $b = 4$, $c = 5$ and $d = 7$ "*abcd*" = 3457. The speech marks mean it's not algebra, so do not multiply them all together.

Examples

Write this number in expanded form: "87*a*2" Answer is $8 \times 1000 + 7 \times 100 + a \times 10 + 2$

Write this number in expanded form: "6*b*370" Answer is $6 \times 10\,000 + b \times 1000 + 3 \times 100 + 7 \times 10$

(a) Write this number in expanded form: "4*a*2" $4 \times 100 + a \times 10 + 2$ ✓✓

(b) Write this number in expanded form: "8*b*12" $8 \times 1000 + b \times 100 + 1 \times 10 + 2$ ✓✓

(c) Write this number in expanded form: "*cd*042" $c \times 10000 + d \times 1000 + 4 \times 10 + 2$ ✓✓

(d) Write this number in expanded form: "*pqrs*" $p \times 1000 + q \times 100 + r \times 10 + s$ ✓✓

3. [3 marks] To add numbers manually it is helpful to rewrite them in columns.

Example: 456 + 78 + 112 + 50 becomes

thousands	hundreds	tens	units
	4	5	6
		7	8
	1	1	2
		5	0

Example: 3125 + 3 + 1101 + 72 becomes

thousands	hundreds	tens	units
3	1	2	5
			3
1	1	0	1
		7	2

(a) Complete the columns for

53 + 741 + 3007 + 4

thousands	hundreds	tens	units
		5	3
	7	4	1
3	0	0	7
			4

✓

(b) Complete the columns for

3000 + 3026 + 8 + 54

thousands	hundreds	tens	units
3	0	0	0
3	0	2	6
			8
		5	4

✓

(c) Complete the columns for

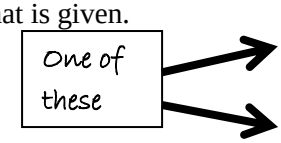
940 + 28 + 1010 + "*abc*"

where in "*abc*" each letter stands for a digit (as in Question 2).

thousands	hundreds	tens	units
	9	4	0
		2	8
1	0	1	0
	a	b	c

✓

4. [15 marks] These additions are wrong! For each one, find the correct total and also the difference between the correct total and the wrong total that is given.



542	28519	39 ✓	15	20944751
106 ✓	66742 ✓	6084	468 ✓	67201912
702	695765	277	95	6829103
		7292	771	95038766
			28	
			623	
			192 ✓	
			2356	
Correct:	648 ✓	695945 ✓	6752 ✓	2392 ✓	49757667 ✓
Difference:	54 ✓	180 ✓	540 ✓	36 ✓	63000 ✓

In each addition, highlight the two digits that were transposed to cause the mistake.
Circle them or use a highlighter.

5. [3 marks] If you were doing a problem like the ones in question 1 and the difference between the correct total and the wrong total was 45, where would you look for transposed digits and what digits might they be? Explain, giving an example if that helps.

$45 = 9 \times 5$ ✓

The transposed digits differ by 5. ✓

The transposed digits will be in the tens and units columns. ✓

6. [3 marks] If you were doing a problem like the ones in question 1 and the difference between the correct total and the wrong total was 720, where would you look for transposed digits and what digits might they be? Explain, giving an example if that helps.

$720 = 9 \times 8 \times 10$ ✓

The transposed digits differ by 8. ✓

The transposed digits will be in the hundreds and tens columns. ✓

7. [10 marks] Simplify these algebraic expressions.

Example: $5x - 2x = 3x$

Example: $100y - 20y = 80y$

Example: $4x - 7x = -3x$

(a) $6x - 4x$ 2x ✓

(b) $10x - 3x$ 7x ✓

(c) $50x - x$ 49x ✓

(d) $2y - y$ y ✓

(e) $6y - 8y$ -2y ✓

(f) $10y - 100y$ -90y ✓

(g) $100y - 10y$ 90y ✓

(h) $x - 10x$ -9x ✓

(i) $1000x - 100x$ 900x ✓

(j) $1000y - 10\,000y$ -9000y ✓

8. [2 marks] Complete this table. (If necessary, divide the numbers by 9 on your calculator.)

		It is a multiple of 9	It is not a multiple of 9
Example	36	✓	
Example	51		✓
Example	900	✓	
	18	✓	
	270	✓	
	458		✓
	90 000	✓	
	77		✓
	63	✓	
	9	✓	
	1		✓

Marks for this table: ✓✓

9. [2 marks] In this question each letter stands for a digit. Simplify these algebraic expressions, and indicate if your answer is a multiple of 9 or not.

	Expression	Simplified	It is a multiple of 9	It is not a multiple of 9
Example	$28a - a$	$27a$	✓	
Example	$11b - 5b$	$6b$		✓
Example	$95c - 5c$	$90c$	✓	
	$19d - d$	18d	✓	
	$48e - 3e$	45e	✓	
	$60f - 4f$	56f		✓
	$901g - g$	900g	✓	
	$100h - 3h$	97h		✓
	$50i - 5i$	45i	✓	
	$3000j - 3j$	2997j	✓	
	$1000k - 100k$	900k	✓	
	$12m - 30m$	-18m	✓	
	$600n - 700n$	-100n		✓
	$10p - 100p$	-90p	✓	

Marks for this table: ✓✓

10. [8 marks] Complete these tables. In this question each letter stands for a digit.

Example	Decimal Form	Expanded form
Number	65432	$6 \times 10\,000 + 5 \times 1000 + 4 \times 100 + 3 \times 10 + 2$
Number with two digits transposed	65342	$6 \times 10\,000 + 5 \times 1000 + 3 \times 100 + 4 \times 10 + 2$
Difference	90	$430 - 340 = 90$

Example	Decimal Form	Expanded form
Number	"861*a*"	$8 \times 1000 + 6 \times 100 + 1 \times 10 + a$
Number with two digits transposed	"681*a*"	$6 \times 1000 + 8 \times 100 + 1 \times 10 + a$
Difference	1800	$8600 - 6800 = 1800$

	Decimal Form	Expanded form
Number	963*b*	$9 \times 1000 + 6 \times 100 + 3 \times 10 + b$
Number with two digits transposed	936*b*	$9 \times 1000 + 3 \times 100 + 6 \times 10 + b$
Difference		$600 - 300 + 30 - 60 = 270$

✓✓

	Decimal Form	Expanded form
Number	4*c*27	$4 \times 1000 + c \times 100 + 2 \times 10 + 7$
Number with two digits transposed	42*c*7	$4 \times 1000 + 2 \times 100 + c \times 10 + 7$
Difference		$c \times 100 - 200 + 20 - c \times 10 = 90c - 180$

✓✓

	Decimal Form	Expanded form
Number	3*def*	$3 \times 1000 + d \times 100 + e \times 10 + f$
Number with two digits transposed	3*edf*	$3 \times 1000 + e \times 100 + d \times 10 + f$
Difference		$100d + 10e - 100e - 10d = 90d - 90e = 90(d - e)$

✓✓

	Decimal Form	Expanded form
Number	*ghij*	$g \times 1000 + h \times 100 + i \times 10 + j$
Number with two digits transposed	*ghji*	$g \times 1000 + h \times 100 + j \times 10 + i$
Difference		$10i + j - 10j - i = 9i - 9j = 9(i - j)$

✓✓