Geometric Proof Test

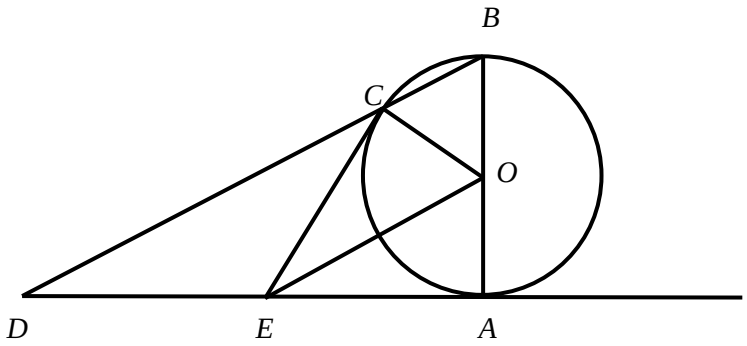
Total = 28 marks. Suggested time = 60 minutes.

1. [4 marks] Triangle *ABC* is isosceles. Given that the size of $\angle ABC$ is $\theta°$

(a) Sketch the two distinct situations that can arise for triangle *ABC*

(b) Prove that the size of $\angle BCA$ for one situation is $\frac{180-\theta}{2}$ and for the other situation the size of $\angle BCA$ is $180-2\theta$

2. [7 marks]



The diagram shows a circle centred at *O* with diameter $\overline{AB}$ and tangents $\overline{EC}$ and $\overline{DA}$.

Complete the seven blank Explanation cells in the tables below.

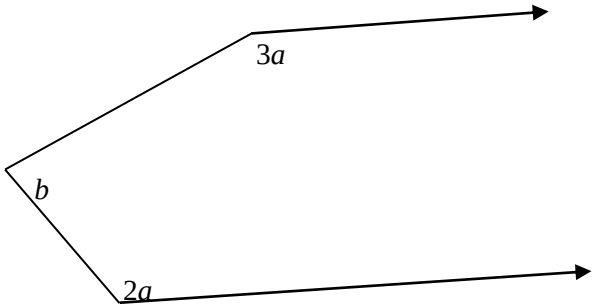
(a) Required to prove: triangle OAE is congruent to triangle OCE.

Statement	**Explanation**
$\overline{OA} \cong \overline{OC}$	Both are radii.
$\angle OAE \cong \angle OCE$	
(Note that ΔOAE and ΔOCE share a common side.)	
$\Delta OAE \cong \Delta OCE$	

(b) Required to prove: E is the midpoint of $\overline{DA}$. That is, $\overline{AE} : \overline{AD} = 1 : 2$

Statement	**Explanation**
$\angle AOE \cong \angle COE$	Corresponding parts of congruent shapes, as shown in the table above.
Let the size of $\angle AOE$ and $\angle COE$ be θ.	
$\angle COB = 180 - 2\theta$	
$\angle OCB = \angle OBC = \theta$	ΔOCB is isosceles, and $\angle COB = 180 - 2\theta$
$\overline{OE} \parallel \overline{BC}$	
$\Delta OAE \sim \Delta BAD$	
$\overline{OA} : \overline{AB} = 1 : 2$	
$\overline{AE} : \overline{AD} = 1 : 2$	

3. [4 marks] *Diagram is not drawn to scale.*

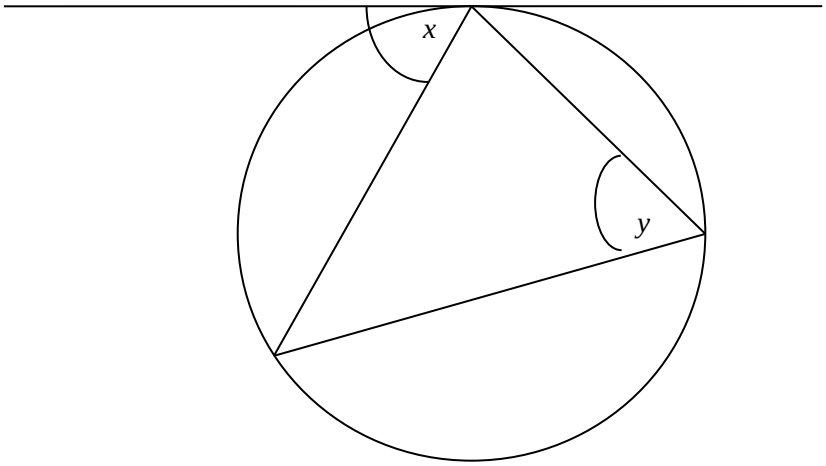


Required to prove: $5a + b = 360°$ (*May not require the use of every row given in the table.*)

Statement	**Explanation**

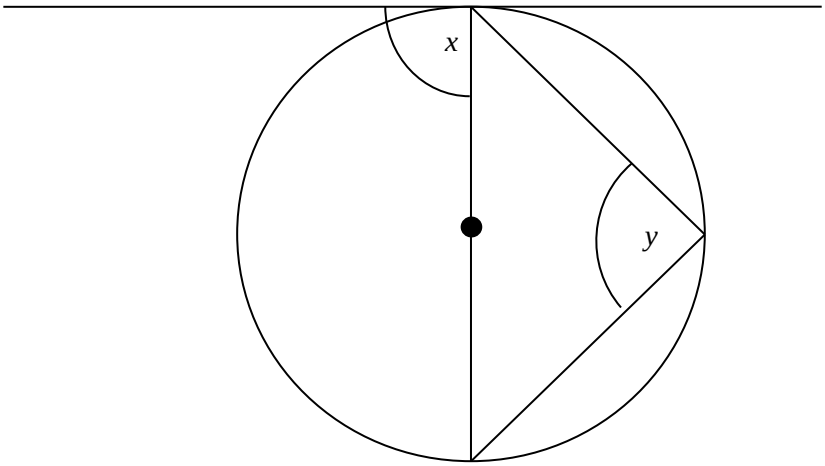
4. [5 marks]

(a) The Alternate Segment Theorem says that for a triangle with a circumcircle and a tangent at one vertex as drawn below, $\angle x = \angle y$. The letters are the sizes of the angles, in degrees.



A student, required to prove the Alternate Segment Theorem, gave this "proof":

Since the AST has to be true for all triangles, it has to be true when one side of the triangle is the diameter of the circle.

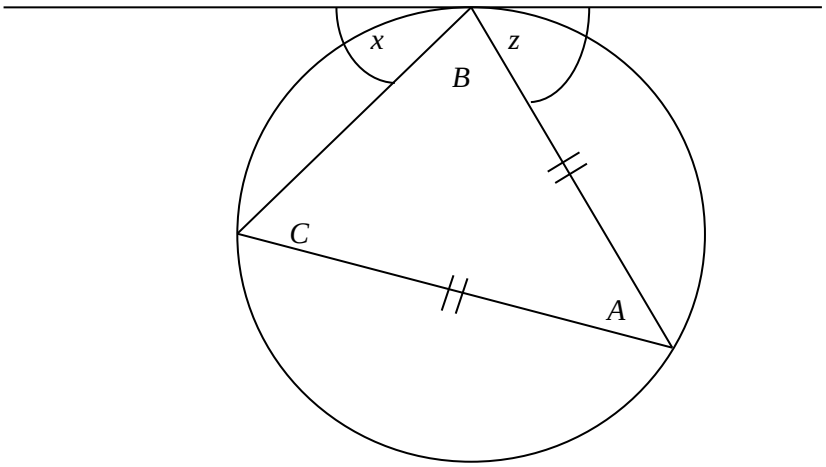


$\angle x = 90°$ because it is the angle between a radius and a tangent. $\angle y = 90°$ because it is the angle from a diameter subtended at the circumference. Therefore, $\angle x = \angle y$.

Explain anything that is wrong with the student's "proof".

(b) This diagram shows an isosceles triangle with its circumcircle and a tangent at one vertex.

The letters are the sizes of the angles, in degrees. Prove that $z = 90 - \frac{x}{2}$

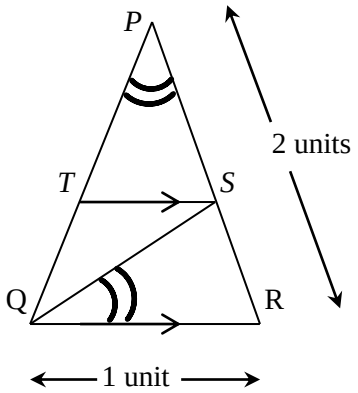


(*May not require the use of every row given in the table. Algebra may be shown below the table.*)

Statement	**Explanation**

5. [4 marks] In the diagram to the right the length $\overline{PR}$ is twice length $\overline{QR}$, $\angle$QPR $\cong$ $\angle$SQR and the lines $\overline{TS}$ and $\overline{QR}$ are parallel.

Diagram is not drawn to scale.



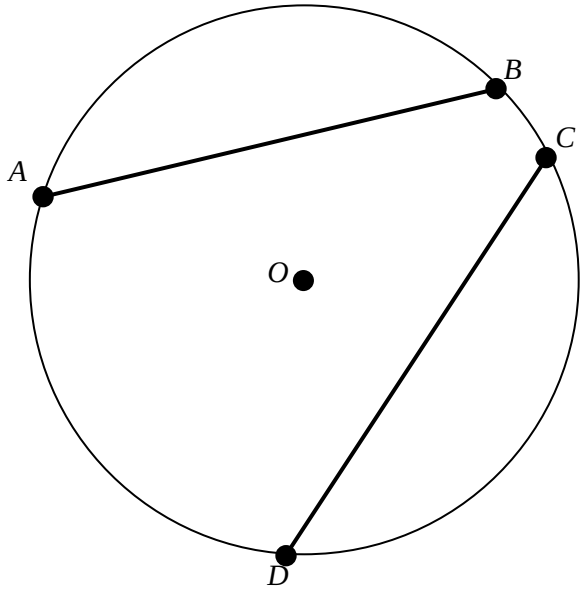
(a) Prove that $\overline{PR}$ is 4 times the length of $\overline{SR}$.

Statement	**Explanation**

(b) Prove that $\overline{SR}$: $\overline{TS}$: $\overline{QR}$ = 2 : 3 : 4

Statement	**Explanation**

6. [4 marks] *Diagram is not drawn to scale.*



The circle centred at *O* has chords $\overline{AB}$ and $\overline{CD}$ which are equidistant from *O*.

(a) Mark the diagram with the shortest lines from *O* to each chord, such that $\overline{OE}$ is the shortest line from *O* to $\overline{AB}$ and $\overline{OF}$ is the shortest line from *O* to $\overline{CD}$. Use appropriate mathematical symbols to record all the known facts of this situation.

(b) Prove that chords $\overline{AB}$ and $\overline{CD}$ are congruent by showing in the table below that $\Delta OAB \cong \Delta OCD$ (*May not require the use of every row given in the table.*)

Statement	**Explanation**