Proof by Induction

Marking Schedule for Test

1. Party Mix Radio [2,2,4 = 8 marks]

(a) $x = 4,\ y = 6$ or $x = 9,\ y = 3$ or $x = 14,\ y = 0$ *one mark for one, full marks for two* ✓✓

(b) The only other times are 4 and 7 minutes. ✓✓

(c) As an extra 3 minute track can be added to any party mix time, once 3 consecutive feasible times are found all greater times are possible. ✓✓

Such times are shown in this table.

x	1	3	0
y	1	0	2
Time	8	9	10

✓✓

2. Powerful cubes [1,2,2,4 = 9 marks]

(a) L.H.S. = R.H.S. = 225 ✓

(b) L.H.S. = R.H.S. = 9 ✓

1. (Zero is not a Natural number.) ✓

(c) The similar fraction is $\frac{(k+1)^2(k+1+1)^2}{4} = \frac{(k+1)^2(k+2)^2}{4} = \frac{[(k+1)(k+2)]^2}{4} = \frac{(k^2+3k+2)^2}{4}$

Any version ✓✓

(d) **Required to prove:** $\mathbf{1^3 + 2^3 + 3^3 + 4^3 + 5^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}}$

Step One

For $n = 1$, L.H.S. = R.H.S. = 1 ✓

Step Two

Assume the equation is true for $n = k$, that is $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + \ldots + k^3 = \frac{k^2(k+1)^2}{4}$ ✓

For $n = k+1$, L.H.S. $= 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + \ldots + k^3 + (k+1)^3 = \frac{k^2(k+1)^2}{4} + (k+1)^3$

$= \frac{(k+1)^2[k^2+4k+4]}{4} = \frac{(k+1)^2(k+2)^2}{4}$ = R.H.S., as shown in part (c) above. ✓✓

3. Counting With Fingers [1,2,2,1,4 = 10 marks]

(a) 35 ✓

(b) $1 + 3(n-1) \;=\; 3n - 2$ *Any correct expression* ✓✓

(c) $\frac{n}{2}[2 + 3(n-1)] \;=\; \frac{n(3n-1)}{2} \;=\; \frac{3n^2-n}{2}$ *Any correct expression* ✓✓

(d) $\frac{k+1}{2}[3(k+1) - 1] \;=\; \frac{1}{2}(k+1)(3k+2) \;=\; \frac{3k^2+5k+2}{2}$ *Any correct expression* ✓

(e) **Required to prove:** $1 + 4 + 7 + 10 + \;...$ for n terms $= \frac{n}{2}[2 + 3(n-1)] \;=\; \frac{n(3n-1)}{2}$

That is, $1 + 4 + 7 + 10 + \;... + (\,3n - 2) \;=\; \frac{n(3n-1)}{2}$

Step One

For $n = 1$, L.H.S. = R.H.S. = 1 ✓

Step Two

Assume the equation is true for $n = k$, that is $1 + 4 + 7 + 10 + \;... + \;(3k-2) \;=\; \frac{k(3k-1)}{2}$ ✓

For $n = k + 1$, L.H.S. = $1 + 4 + 7 + 10 + \;... + \;(3k-2) + \;(3(k+1) - 2)$

$= \frac{k(3k-1)}{2} \;+\; 3(k+1) - 2 \;=\; \frac{k(3k-1)}{2} \;+\; 3k + 1$

$= \frac{k(3k-1) + 6k + 2}{2} \;=\; \frac{3k^2+5k+2}{2}$ = R.H.S., as shown in part (d) above. ✓✓

4. A False Proof [3 marks]

There is nothing wrong with Step One, except that it verifies only one case.

Ordinary induction requires that only the first case needs to be verified, but in "strong induction" – as in this example – more than one case must be verified.

The assumption made in the proof is:
"Assume the equation is correct for all $n \le k$, that is $2^0 = 1$, ... , $2^{k-1} = 1$, $2^k = 1$."
This assumption contains an ellipsis mark where the statements $2^1 = 1$, $2^2 = 1$, $2^3 = 1$, fit. These statements are simply incorrect and cannot be validly "assumed".

Correct explanation ✓✓
Well expressed ✓

The false proofs given in this assignment are modified from the collection of false induction proofs at *http://www.math.uiuc.edu/~hildebr/347honors/induction4.pdf*